

# Effective Gauge Theories with Symmetry Breaking

G. Grimm

Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen

Z. Naturforsch. **49a**, 1093–1101 (1994); received August 26, 1994

In a previous paper a general group theoretical discussion of effective boson-fermion coupling theories (with composite two-fermion bosons) resulting by weak mapping of nonlinear spinor-isospinor field models was given. In the present work these considerations are extended to the case of broken isospin symmetry and applied to a microscopic subfermion-description of phenomenological gauge theories with Higgs symmetry breaking.

PACS 11.10 – Field theory.

PACS 12.10 – Unified field theories and models.

PACS 12.35 – Composite models of particles.

## Introduction

Present high energy physics is dominated by the concept of (local) gauge invariance [1]. Interactions between particles are commonly described by exchange of gauge bosons. In particular, the application of Yang-Mills theories to electroweak and strong interactions was remarkably successful [2–7]. At present, the gauge theories of electroweak and strong interactions constitute the generally adopted standard model of elementary particles. In this model, quarks and leptons are considered as the basic constituents of matter, interacting via gauge bosons. Their empirical properties are well-described by the well-known parametrization scheme of families and generations. Furthermore, this model has the advantage of being renormalizable, i.e. the successful renormalization program of QED can be applied to obtain finite results. However, in contrast to QED, the finite range of weak and strong interaction implies massive gauge bosons which would destroy renormalizability. This problem could be circumvented by the concept of spontaneous symmetry breaking which allows mass generation without spoiling renormalizability [8, 9]. To this purpose an additional self-interacting scalar field, the Higgs boson field, was introduced which leads to mass terms due to non-vanishing vacuum expectation values. This Higgs field, however, not only increases the number of unexplained structures and parameters in the standard model, but also leads to serious conceptional problems, in particular in con-

nection with grand unified theories (see for example [10]).

Therefore, several attempts were made to replace the Higgs mechanism by some kind of dynamical symmetry breaking. First, Weinberg discussed physical implications of dynamical symmetry breaking within the standard model [11, 12]. These ideas led to the technicolor model as a substitute for the standard Higgs mechanism [10, 13]. While in these models (fictive) gauge interactions were considered as dynamical origin of symmetry breaking, in subsequent models the Higgs sector was replaced by a nonlinear self-interaction which was added to the gauge couplings [14, 15].

However, in a model with fundamental gauge interaction an additional nonlinear self-interaction appears as artificial admixture. On the other hand, there are subfermion models with fundamental nonlinear self-interactions as Heisenbergs spinor theory [16] or related models [17, 18]. They contain the possibility of symmetry breaking by nonlinear interactions from the outset. For example Nambu and Jona-Lasinio [19, 20] used a nonlinear spinor model of the Heisenberg type to demonstrate chiral symmetry breaking by condensates in analogy to the BCS theory of superconductivity. Thus, if gauge interactions are considered as effective interactions of composite particles, these models provide a natural tool for a self-contained treatment of the standard model with effective gauge interactions and dynamical symmetry breaking.

Essential features of such subfermionic descriptions shall be considered in this paper. We follow the treatment of [21] where a general group theoretical discus-

Reprint requests to Dr. G. Grimm.

0932-0784 / 94 / 1200-1093 \$ 06.00 © – Verlag der Zeitschrift für Naturforschung, D-72027 Tübingen



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

sion of effective boson-fermion coupling theories resulting from underlying nonlinear spinor field models was presented. In particular, it was demonstrated that the field equations of gauge bosons and their couplings to matter fields can be reproduced as effective dynamics. Thus, if the treatment is applied to subfermion models of elementary particles, there is hope to replace the standard gauge theories of electroweak and strong interactions by an effective theory of composite particles. Therefore, we extend our investigations to the case of broken isospin symmetry.

We consider a spinor-isospinor model with local four-fermion interaction and global  $SU(2) \times U(1)$  isospin symmetry. Within this model we describe bosons as two-fermion bound states and represent fermions by the elementary spinor field, leading to an extremely simplified version of the standard model. We do not introduce additional Higgs fields. Rather, we suppose the model to be self-contained. The symmetry breaking results from non-symmetric ground state solutions leading to a non-symmetric propagator. This non-symmetric propagator function leads to modifications in the calculations of the effective boson-fermion theory. The resulting equations correspond to those obtained by the standard Higgs mechanism.

In the first section we will briefly introduce the class of subfermion models under consideration. Then, we describe the weak mapping procedure which is applied to derive the effective dynamics of composite particles. A more detailed discussion is presented in [21] and references cited therein. Here we will give a slightly modified version in order to incorporate the possibility of broken isospin symmetry. In Sect. 3 the effects of isospin symmetry breaking on the effective dynamics are discussed. The most interesting point, namely the resulting effective gauge field equations, is considered in more detail in Section 4.

## 1. Nonlinear Spinor Field Model

We consider a Dirac-spinor-isospinor field  $\psi_{\alpha A}(x)$ ,  $\alpha = 1, \dots, 4$ ,  $A = 1, 2$  with a Lorentz- and isospin- $U(2)$ -invariant local four-fermion self-interaction which is given in an explicitly Fierz-antisymmetrized form. If the isospin symmetry is shared by the kinetic term, the field equation for the spinor-isospinor field  $\psi(x)$  reads

$$(i\gamma^\mu \partial_\mu - m_0)_{\text{reg}} \psi(x) = \sum_h g_h v_h \psi(x) (\bar{\psi}(x) v^h \psi(x)) + \sum_h \tilde{g}_h v_h C \bar{\psi}^T(x) (\psi^T(x) C v^h \psi(x)) \quad (1)$$

with the basic vertex elements

$$v^h = v^{st} \equiv \Gamma^s \otimes \tau^t \in \{\gamma^\mu, \Sigma^{\mu\nu}, \gamma_5 \gamma^\mu, i\gamma_5, \mathbf{1}\} \otimes \{\mathbf{1}, \tau^k\}, \quad h = (s, t)$$

$$v_h = v_{st} \equiv \Gamma_s \otimes \tau^t \in \{\gamma_\mu, \Sigma_{\mu\nu}, \gamma_5 \gamma_\mu, i\gamma_5, \mathbf{1}\} \otimes \{\mathbf{1}, \tau^k\} \quad (2)$$

in the direct product space of spin and isospin and corresponding coupling constants  $g_h$  and  $\tilde{g}_h$ .  $C$  denotes the charge conjugation matrix,  $\{\tau^t, t = 0, \dots, 3\}$  are the Pauli matrices in isospace. A suitable regularization scheme is implicitly assumed but not explicitly applied. For quantitative calculations we refer for example to the auxiliary-field-regularized subfermion model of Stumpf [18, 22–25].

To take into account the possibility of explicit symmetry breaking from the outset, we replace the kinetic operator in (1) with the mass parameter  $m_0$  by

$$K = (i\gamma^\mu \partial_\mu - M)_{\text{reg}} \quad (3)$$

with the mass matrix

$$M = m_1 \frac{1 + \tau^3}{2} + m_2 \frac{1 - \tau^3}{2} = m_0 (\mu_0 \mathbf{1} + \mu_3 \tau^3) \quad (4)$$

and

$$\mu_0 = \frac{\frac{1}{2}(m_1 + m_2)}{m_0}, \quad \mu_3 = \frac{\frac{1}{2}(m_1 - m_2)}{m_0}. \quad (5)$$

I.e. we assume arbitrary masses  $m_1$  and  $m_2$  for the first and second isospin component. Equivalently, this is expressed by some reference mass  $m_0$  and dimensionless parameters  $\mu_0$  and  $\mu_3$  which reduce to the symmetric case (1) for  $\mu_0 = 1$  and  $\mu_3 = 0$ .

To compactify the notation we introduce a super-spinor  $\psi = (\psi_A)$ ,  $A = 1, 2$  which combines the spinor  $\psi$  and the charge conjugated spinor  $\psi^C = C \bar{\psi}^T$  by the definitions  $\psi_{A=1} := \psi$ ,  $\psi_{A=2} := \psi^C$ . Then, denoting Pauli matrices in superspace by  $\{\lambda^l, l = 0, \dots, 3\}$  the field equation (1) and its charge conjugated counterpart can be combined into the equation

$$K \psi(x) = \frac{1}{2} \sum_H g_H v_H \psi(x) (\psi^T(x) w^H \psi(x)), \quad (6)$$

with

$$v_H = v_{stl} := v_h \lambda^l = \Gamma_s \tau^t \lambda^l, \quad w^H = w^{stl} := C \lambda^l v^{stl} = C \Gamma^s \tau^t \lambda^l \quad (7)$$

and

$$g_{st0} = g_{st3} := g_{st}, \quad g_{st1} = g_{st2} := \tilde{g}_{st}. \quad (8)$$

Finally in a superindex notation with  $I = (Z, x) = (\alpha, A, \lambda, x)$  and an extension of the summation rule to integrations for continuous indices, the field equations (6) read

$$K_{I_1 I} \psi_I = V_{I_1 I_2 I_3 I_4} \psi_{I_2} \psi_{I_3} \psi_{I_4} \quad (9)$$

with

$$\begin{aligned} K_{I_2 I_2} &:= K_{Z_1 Z_2}(x_1) \delta(x_1 - x_2) \\ &:= (D_\mu^{\mu(1)} - M)_{Z_1 Z_2/\text{reg}} \delta(x_1 - x_2), \\ D_{Z_1 Z_2}^\mu &:= i \gamma_{\alpha_1 \alpha_2}^\mu \delta_{A_1 A_2} \delta_{\lambda_1 \lambda_2}, \\ M_{Z_1 Z_2} &:= m_0 (\mu_0 \mathbf{1} + \mu_3 \tau^3)_{A_1 A_2} \delta_{\alpha_1 \alpha_2} \delta_{\lambda_1 \lambda_2} \end{aligned} \quad (10)$$

and

$$\begin{aligned} V_{I_1 I_2 I_3 I_4} &:= V_{Z_1 Z_2 Z_3 Z_4} \delta(x_1 - x_2) \delta(x_1 - x_3) \delta(x_1 - x_4), \\ V_{Z_1 Z_2 Z_3 Z_4} &:= \frac{1}{2} \sum_H g_H(\mathbf{v}_H)_{Z_1 Z_2} (\mathbf{w}^H)_{Z_3 Z_4}. \end{aligned} \quad (11)$$

The spinor quantum field theory which is defined by these field equations and the anti-commutation relations  $\{\psi_I, \psi_{I'}\} = A_{II'}$  can be described in terms of functional states and corresponding functional equations [26]. In the weak mapping procedure, these functional equations are formulated in an energy representation for equal times [26, 27]. The generating functional state for normal ordered matrix elements  $\varphi_n(I_1 \dots I_n | \alpha) = \langle \Omega | \mathcal{N} \{ \psi_{I_1} \dots \psi_{I_n} \} | \alpha \rangle$  corresponding to some quantum state  $|\alpha\rangle$  reads

$$|\mathcal{F}[j, a]\rangle = \sum_n \frac{i^n}{n!} \varphi_n(I_1 \dots I_n | a) j_{I_1} \dots j_{I_n} |0\rangle \quad (12)$$

with  $I = (Z, x)$  and anticommuting sources  $j_I$  which act in an auxiliary functional Fock space with vacuum state  $|0\rangle$ . The corresponding functional energy equation reads

$$\begin{aligned} (E_a - E_0) |\mathcal{F}[j, a]\rangle &= j_{I_1} \left\{ \tilde{K}_{I_1 I_2} \frac{\delta}{\delta j_{I_2}} + W_{I_1 I_2 I_3 I_4} \left[ \frac{\delta}{\delta j_{I_4}} \frac{\delta}{\delta j_{I_3}} \frac{\delta}{\delta j_{I_2}} - 3 F_{I_4 I_4} j_{I_4} \frac{\delta}{\delta j_{I_3}} \frac{\delta}{\delta j_{I_2}} + (3 F_{I_4 I_4} F_{I_3 I_3} \right. \right. \\ &\quad \left. \left. + \frac{1}{4} A_{I_4 I_4} A_{I_3 I_3} \right) j_{I_4} j_{I_3} \frac{\delta}{\delta j_{I_2}} - (F_{I_4 I_4} F_{I_3 I_3} \right. \\ &\quad \left. \left. + \frac{1}{4} A_{I_4 I_4} A_{I_3 I_3} \right) F_{I_2 I_2} j_{I_4} j_{I_3} j_{I_2} \right\} |\mathcal{F}[j, a]\rangle \\ &:= \mathcal{H} \left[ j, \frac{\delta}{\delta j} \right] |\mathcal{F}[j, a]\rangle \end{aligned} \quad (13)$$

with

$$\begin{aligned} \tilde{K}_{I_1 I_2} &:= (\tilde{\gamma}^k \partial_k^1 + \tilde{M})_{Z_1 Z_2} \delta(x_1 - x_2), \quad \tilde{\gamma}^k \partial_k + \tilde{M} := -\gamma^0 (i \gamma^k \partial_k - M), \\ W_{I_1 I_2 I_3 I_4} &:= W_{Z_1 Z_2 Z_3 Z_4} \delta(x_1 - x_2) \delta(x_1 - x_3) \delta(x_1 - x_4), \\ W_{Z_1 Z_2 Z_3 Z_4} &:= \frac{1}{2} \sum_H g_H(\mathbf{u}_H)_{Z_1 Z_2} \mathbf{w}_{Z_3 Z_4}^H, \quad \mathbf{u}_H := -\gamma^0 \mathbf{v}_H. \end{aligned} \quad (14)$$

$F_{I_1 I_2} := F_{Z_1 Z_2}(x_1 - x_2)$  denotes the equal time limit of the propagator, and  $E_0$  the ground state energy. The functional Hamiltonian  $\mathcal{H} \left[ j, \frac{\delta}{\delta j} \right]$  completely characterizes the spinor quantum field theory.

## 2. Effective Composite Particle Dynamics

The derivation of composite particle dynamics by weak mapping of the functional equation (13) is described in detail in [26, 27]. In [28], by means of this method, an effective boson-fermion dynamics for two-particle bosons and elementary fermions was derived. This treatment will be adopted here.

At first, a complete set of two-particle wave functions is introduced, characterized by a bosonic index  $K$  and denoted by  $\{\mathcal{C}_K^{I_1 I_2}\}$ . With this set and corresponding dual states  $\{\mathcal{D}_{I_1 I_2}^K\}$  the functional states (12) can be reformulated as boson-fermion functional states:

$$\begin{aligned} |\mathcal{F}[j, a]\rangle &= \sum_{mn} \frac{i^m}{m!} \frac{i^n}{n!} \varrho(K_1 \dots K_m I_1 \dots I_n | a) \\ &\quad \cdot b_{K_1} \dots b_{K_m} j_{I_1} \dots j_{I_n} |0\rangle := |\tilde{\mathcal{F}}[j, b; a]\rangle \end{aligned} \quad (15)$$

(with commuting bosonic sources  $b_K$ ), and from (13) the effective boson-fermion functional equation

$$\mathcal{H} \left[ j, \frac{\delta}{\delta j}, b, \frac{\delta}{\delta b} \right] |\tilde{\mathcal{F}}[j, b; a]\rangle = (E_a - E_0) |\tilde{\mathcal{F}}[j, b; a]\rangle \quad (16)$$

can be derived. According to [21] we use a leading-term-approximation corresponding to a classical low-energy limit. In this case the effective functional

Hamiltonian reads [28]

$$\begin{aligned}
 \tilde{\mathcal{H}} \left[ j, \frac{\delta}{\delta j}, b, \frac{\delta}{\delta b} \right] &= [2 \mathcal{R}_{I_1 I_2}^{K_1} \tilde{K}_{I_1 I_2} \mathcal{C}_{K_2}^{I_2 I} + 6 W_{I_1 I_2 I_3 I_4} F_{I_2 I} \mathcal{R}_{I_1 I}^{K_1} \mathcal{C}_{K_2}^{I_4 I_3}] b_{K_1} \frac{\delta}{\delta b_{K_2}} \\
 &\quad + 12 W_{I_1 I_2 I_3 I_4} \mathcal{R}_{I_1 I}^{K_3} \mathcal{C}_{K_1}^{I_3 I_4} \mathcal{C}_{K_2}^{I_2 I} b_{K_3} \frac{\delta}{\delta b_{K_2}} \frac{\delta}{\delta b_{K_1}} \\
 &\quad + 6 W_{I_1 I_2 I_3 I_4} \mathcal{C}_{K_1}^{I_3 I_4} j_{I_1} \frac{\delta}{\delta j_{I_2}} \frac{\delta}{\delta b_{K_1}} + 3 W_{I_1 I_2 I_3 I_4} F_{I_2 I} \mathcal{R}_{I_1 I}^{K_1} b_{K_1} \frac{\delta}{\delta j_{I_3}} \frac{\delta}{\delta j_{I_4}} \\
 &\quad + \tilde{K}_{I_1 I_2} j_{I_1} \frac{\delta}{\delta j_{I_2}} + W_{I_1 I_2 I_3 I_4} j_{I_1} \frac{\delta}{\delta j_{I_4}} \frac{\delta}{\delta j_{I_3}} \frac{\delta}{\delta j_{I_2}} \\
 &:= \mathcal{H}_b \left[ b, \frac{\delta}{\delta b} \right] + \mathcal{H}_{b-f} \left[ j, \frac{\delta}{\delta j}, b, \frac{\delta}{\delta b} \right] + \mathcal{H}_f \left[ j, \frac{\delta}{\delta j} \right]. \quad (17)
 \end{aligned}$$

This functional Hamiltonian describes the effective boson-fermion dynamics on the functional level. It consists of a pure bosonic part, a boson-fermion-coupling part, and a pure fermionic part.

For the further evaluation of the effective dynamics explicit expressions for the composite-particle amplitudes  $\mathcal{C}_K$  resp.  $\mathcal{R}^K$  and the propagator  $F$  have to be inserted. Again, we follow the general group theoretical treatment in low energy approximation discussed in [21], with modifications due to symmetry breaking. For the composite-particle amplitudes (which shall be restricted to bound states) we use the expression

$$\begin{aligned}
 \mathcal{C}_{I_1 I_2}^K &= \mathcal{C}_{Z_1 Z_2}^{stl} e^{i\mathbf{k} \cdot \mathbf{z}} \chi(\mathbf{u}|\mathbf{k}) \\
 &= \mathcal{S}_{\alpha_1 \alpha_2}^s \mathcal{T}_{A_1 A_2}^t \mathcal{L}_{\Lambda_1 \Lambda_2}^l e^{i\mathbf{k} \cdot \mathbf{z}} \chi(\mathbf{u}|\mathbf{k}), \quad (18)
 \end{aligned}$$

where the bosonic states are classified by the index  $K = (s, t, l, \mathbf{k})$  which expresses the properties corresponding to spin, isospin, superspin and center-of-mass momentum. The algebraic part is explicitly given by

$$\{\mathcal{C}^{stl}\} = \{\mathcal{S}^s, s = 1 \dots 16\} \otimes \{\mathcal{T}^t, t = 0 \dots 3\} \otimes \{\mathcal{L}^l, l = 0 \dots 3\} \quad (19)$$

$$= \{(\Gamma_s)^+ C\} \otimes \{\tau^t\} \otimes \{\lambda^l \lambda^1\}. \quad (20)$$

The orbital part is restricted to ground states (neglecting internal excitations) with symmetric  $s$  wave functions

$$\chi(-\mathbf{u}|\mathbf{k}) = \chi(\mathbf{u}|\mathbf{k}) \approx \chi(\mathbf{u}|0) := \chi(\mathbf{u}) \quad (21)$$

where the dependence on the momentum  $\mathbf{k}$  is neglected. Due to this symmetric orbital part the anti-symmetry of the amplitudes  $\mathcal{C}_{I_2 I_1}^K = -\mathcal{C}_{I_1 I_2}^K$  is reduced to the algebraic part  $\mathcal{C}^{stl}$ . The dual states are given by  $\mathcal{R}_{I_1 I_2}^{K_1} = \frac{1}{16} (\mathcal{C}_{I_1 I_2}^{K_1})^*$ .

For the translation of the functional equations into field equations it has to be observed that the composite particle states from (18) correspond to bosonic fields  $\Phi_K$ . For these fields we use the designations

$$\begin{aligned}
 \{\Phi_K\} &= \{\Phi_{stl}(\mathbf{z})\} \\
 &= \{A_\mu^{tl}(\mathbf{z}), F_{\mu\nu}^{tl}(\mathbf{z}), G_\mu^{tl}(\mathbf{z}), \phi^{tl}(\mathbf{z}), Z^{tl}(\mathbf{z})\}, \quad (22)
 \end{aligned}$$

which explicitly express the Lorentz properties ( $A_\mu$ : vector,  $F_{\mu\nu}$ : tensor,  $G_\mu$ : axialvector,  $\phi$ : pseudoscalar,  $Z$ : scalar). The isospin properties are expressed by the index  $t$  in the following way:  $t = 0$ : isosinglet (isospin  $T = 0$ ),  $t = 1, 2, 3$ : isotriplet (isospin  $T = 1$ ). Finally, the superspin properties corresponding to fermion number are expressed by the value of  $l$ :

$$\begin{aligned}
 l = 0: & \{(\psi\psi^C + \psi^C\psi)\text{-amplitudes}\} \text{ fermion number } f = 0, \\
 l = 3: & \{(\psi\psi^C + \psi^C\psi)\text{-amplitudes}\} f = 0, \\
 l = 1: & \{(\psi\psi - \psi^C\psi^C)\text{-amplitudes}\} \text{ fermion number } f = \pm 2 \\
 l = 2: & \{\frac{1}{i}(\psi\psi + \psi^C\psi^C)\text{-amplitudes}\} f = \pm 2
 \end{aligned}$$

This purely algebraic ansatz for the amplitudes is independent of any symmetry breaking effects. The implications of symmetry breaking will be sketched in the following.

### 3. Effective Boson-Fermion Dynamics with Isospin Symmetry Breaking

In a nonlinear theory symmetry breaking solutions appear quite naturally (see for example the spontaneous magnetization of a ferromagnet). If the isospin symmetry is not broken spontaneously, the ground states  $\Omega$  carries no isospin charge and the propagator has the form of an isosinglet:  $\langle \psi \bar{\psi} \rangle_\Omega \sim \mathbf{1}_r$  (expressing

identical vacuum properties of both isospin components). If the isospin symmetry is, however, not shared by the ground state, the propagator will be asymmetrical:  $\langle \psi \bar{\psi} \rangle_\Omega \sim \mathbf{1}_\tau + \alpha \tau^3$ . As we will not deal with the dynamical determination of the ground state solution (which should in principle be possible) we will use a general ansatz for the propagator function that incorporates the possibility of broken isospin symmetry:

$$F(x) = \left[ f_1(x) \frac{1+\tau^3}{2} + f_2(x) \frac{1-\tau^3}{2} \right] \lambda^1 = F(x) \lambda^1. \quad (23)$$

Here  $\lambda^1$  expresses the superspin dependence (fermion-antifermion) while  $F(x)$  contains the spin-isospin part. Denoting the spinor doublet by  $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$ ,  $f_1(x)$  represents the  $p$ -propagator and  $f_2(x)$  the  $n$ -propagator. Using the equivalent form

$$F(x) = \frac{1}{2} [f_1(x) + f_2(x)] \mathbf{1}_\tau + \frac{1}{2} [f_1(x) - f_2(x)] \tau^3 \\ =: f_0(x) \mathbf{1}_\tau + f_3(x) \tau^3, \quad (24)$$

We start with the first term which is given by

$$\mathcal{H}_0 = 2b_{K_1} \mathcal{R}_{I_1 I}^{K_1} \tilde{K}_{I_1 I_2} \mathcal{C}_{K_2}^{I_2 I} \frac{\delta}{\delta b_{K_2}}. \quad (25)$$

The evaluation according to [21] leads to the following result

$$\mathcal{H}_0 = \int d^3 z b^{H_1}(z) [\text{Tr}_1(H_1 H_2, k) \partial_k + 2 \text{Tr}_2(H_1 H_2)] \frac{\delta}{\delta b^{H_2}}(z) \quad (26)$$

with

$$\text{Tr}_1(H_1 H_2, k) = \text{Tr}_1(s_1 t_1 l_1 s_2 t_2 l_2, k) = -i \text{tr} \left\{ \left( \frac{1}{4} \mathcal{S}^{s_1} \right)^+ \gamma^0 \gamma^k \mathcal{S}^{s_2} \right\} \delta^{t_1 t_2} \delta^{l_1 l_2} \\ =: \text{Tr}_1(s_1, s_2, k) \quad (27)$$

and

$$\text{Tr}_2(H_1 H_2) = \text{Tr}_2(s_1 t_1 l_1 s_2 t_2 l_2) = \text{tr} \left\{ \left( \frac{1}{4} \mathcal{S}^{s_1} \right)^+ \gamma^0 \mathcal{S}^{s_2} \right\} \text{tr} \left\{ \left( \frac{1}{2} \mathcal{T}^{t_1} \right)^+ M \mathcal{T}^{t_2} \right\} \delta^{l_1 l_2} \\ =: \text{Tr}_2(s_1, s_2) = m_0 \text{Tr}_2(t_1 t_2) \quad (28)$$

The modifications due to the explicit symmetry breaking are expressed by the trace

$$\text{Tr}_2(t_1 t_2) := \frac{1}{2} \text{tr} \{ \tau^{t_1} (\mu_0 \mathbf{1} + \mu_3 \tau^3) \tau^{t_2} \}, \quad (29)$$

which reduces to  $\text{Tr}_2(t_1 t_2) = \delta^{t_1 t_2}$  for  $\mu_3 = 0$ . With (27), (28) and (29) the kinematic functional Hamiltonian (26) reads

$$\mathcal{H}_0 = \int d^3 z b^{s_1 t_1 l_1}(z) [-i \text{Tr}_1(s_1, s_2, k) \delta^{t_1 t_2} \partial_k + \bar{m} \text{Tr}_2(s_1, s_2) \text{Tr}_2(t_1 t_2)] \frac{\delta}{\delta b^{s_2 t_2 l_2}}(z) \quad (30)$$

with  $\bar{m} = 2m_0$ .

Now we consider the second term in (17) which is given by

$$\mathcal{H}_1 = 6b_{K_1} W_{I_1 I_2 I_3 I_4} F_{I_2 I} \mathcal{R}_{I_1 I}^{K_1} \mathcal{C}_{K_2}^{I_4 I_3} \frac{\delta}{\delta b_{K_2}}. \quad (31)$$

Direct evaluation yields

$$\mathcal{H}_1 = -48 \chi(0) g_{H_2} \int d^3 k_2 d^3 z b^{H_1}(z) \underbrace{\int d^3 u \text{tr} \{ \mathcal{R}_{H_1}^T u^{H_2} F(u) \} \chi^*(u) e^{i k_2 \cdot u/2} e^{i k_2 \cdot z}}_{=: \text{Tr}_3(H_1 H_2 | k_2)} \frac{\delta}{\delta b^{H_2}}(k_2). \quad (32)$$

the symmetry breaking term proportional to  $\tau^3$  becomes obvious. Without symmetry breaking it is  $f_1(x) = f_2(x)$ , i.e.  $f_3(x) = 0$ .

Besides the propagator, also the mass term will be affected by symmetry breaking. Moreover, kinetic operator and propagator are closely related by renormalization. We will not give a quantitative discussion of this point. Instead, we use the general expression (4) for the mass term.

Now we can evaluate the effective dynamics given by the functional Hamiltonian (17). As the effects of symmetry breaking are only manifested in the kinetic operator  $\tilde{K}$  and the propagator  $F$ , only those terms which depend on these quantities are modified compared to the treatment in [21]. If, in a low energy approximation, the influence of the symmetry breaking on the fermion-boson coupling is neglected, only the kinetic part of the effective bosonic theory has to be considered. This part is given by the first line in (17).



With (23) for the propagator it follows for the trace:

$$\begin{aligned} \text{Tr}_3(H_1 H_2 | \mathbf{k}) = & -\frac{1}{8} \delta^{l_1 l_2} \int d^3 u \chi^*(\mathbf{u}) e^{i \mathbf{k}_2 \cdot \mathbf{u}/2} [\text{tr} \{(\mathcal{S}^{s_1})^+ \gamma^0 \Gamma^{s_2} f_0(\mathbf{u})\} 2 \delta^{t_1 t_2} \\ & + \text{tr} \{(\mathcal{S}^{s_1})^+ \gamma^0 \Gamma^{s_2} f_3(\mathbf{u})\} \text{tr} \{\tau^{t_1} \tau^{t_2} \tau^{t_3}\}] \end{aligned} \quad (33)$$

which can be approximated for low energies by

$$\text{Tr}_3(H_1 H_2 | \mathbf{k}) \simeq \text{Tr}_3(H_1 H_2) = -c_f \eta_{s_2} \text{Tr}_2(s_1 s_2) \text{Tr}_3(t_1 t_2) \delta^{l_1 l_2} \quad (34)$$

with

$$\text{Tr}_3(t_1 t_2) := \frac{1}{2} \text{tr} \{(\alpha_0 \mathbf{1} + \alpha_3 \tau^3) \tau^{t_1} \tau^{t_2}\} \quad (35)$$

and

$$\eta_s = \begin{cases} 1 & \text{for } \mathcal{S}^s \in \{\mu^\mu, \Sigma^{\mu\nu}, \mathbf{1}\} \\ -1 & \text{for } \mathcal{S}^s \in \{\gamma_5 \gamma^\mu, -i \gamma_5\}. \end{cases} \quad (36)$$

The coefficients  $\alpha_0$  and  $\alpha_3$  depend on the degree of symmetry breaking in the propagator. Without symmetry breaking it is:  $\alpha_0 = 1$ ,  $\alpha_3 = 0$ . With (34) the final result for  $\mathcal{H}_1$  reads

$$\mathcal{H}_1 = c_1 g_{s_2 t_2 l}^* \int d^3 z b^{s_1 t_1 l}(z) \text{Tr}_2(s_1 s_2) \text{Tr}_3(t_1 t_2) \frac{\delta}{\delta b^{s_2 t_2 l}}(z) \quad (37)$$

with  $c_1 = 48 c_f \chi(0)$  and  $g_{s_1 t_1 l}^* = \eta_s g_{s_1 t_1 l}$ .

Together with (30) it results for the kinetic part of the effective functional Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{kin}} = & \int d^3 z b^{s_1 t_1 l}(z) \{ -i \text{Tr}_1(s_1 s_2, k) \delta^{t_1 t_2} \partial_k \\ & + \text{Tr}_2(s_1 s_2) [\bar{m} \text{Tr}_2(t_1 t_2) + \Delta_{s_2 t_2 l} \text{Tr}_3(t_1 t_2)] \} \frac{\delta}{\delta b^{s_2 t_2 l}}(z) \end{aligned} \quad (38)$$

with  $\Delta_{s_1 t_1 l} = c_1 g_{s_1 t_1 l}^*$ . The corresponding field equation reads (compare [21])

$$i \partial_0 \Phi_{s_1 t_1 l}(z) = \{ -i \text{Tr}_1(s_1, s_2, k) \delta_{t_1 t_2} \partial_k + \text{Tr}_2(s_1 s_2) [\bar{m} \text{Tr}_2(t_1 t_2) + \Delta_{s_2 t_2 l} \text{Tr}_3(t_1 t_2)] \} \Phi_{s_2 t_2 l}(z). \quad (39)$$

In this equation, the kinematic field equations for the fields in (22) are summed up. They are supplemented by terms expressing the bosonic self-interaction and the coupling between bosons and fermions which result from the remaining terms of (17). These terms are explicitly quoted in [21] and omitted here for brevity. In the following, the kinematic part (39), which is affected by symmetry breaking, will be further evaluated. As we are particularly interested in composite gauge bosons, we will consider the case of vector bosons.

#### 4. Effective Gauge Field Equations

With the explicit designations for the fields according to (22) the field equations (39) read for vector bosons

$$\begin{aligned} \partial_0 A_0^{t_1 l} &= \partial_k A_k^{t_1 l}, \\ \partial_0 A_k^{t_1 l} &= \partial_k A_0^{t_1 l} + [\bar{m} \text{Tr}_2(t_1 t_2) + \Delta_{T t_2 l} \text{Tr}_3(t_1 t_2)] E_k^{t_2 l}, \\ \partial_0 E_k^{t_1 l} &= -\varepsilon_{klm} \partial_l B_m^{t_1 l} - [\bar{m} \text{Tr}_2(t_1 t_2) \\ &\quad + \Delta_{V t_2 l} \text{Tr}_3(t_1 t_2)] A_k^{t_2 l}, \\ \partial_0 B_k^{t_1 l} &= \varepsilon_{klm} \partial_l E_m^{t_1 l}. \end{aligned} \quad (40)$$

Now we specialize to states with fermion number 0, given by  $l = 0$  or  $l = 3$  depending on the spin-isospin symmetry, and introduce the designations

$$A_\mu^0 \equiv A_\mu^{03}, \quad A_\mu^1 \equiv A_\mu^{13}, \quad A_\mu^2 \equiv A_\mu^{20}, \quad A_\mu^3 \equiv A_\mu^{33} \quad (41)$$

(analogous for the field strengths  $F_{\mu\nu}^{t_l}$ ). Then,  $A_\mu^0$  represents the isosinglet (corresponding to the U(1) boson) and  $A_\mu^i$ ,  $i = 1, 2, 3$  the isotriplet (corresponding to the SU(2) bosons). With the traces  $\text{Tr}_2(t_1 t_2)$  and  $\text{Tr}_3(t_1 t_2)$  from (29) and (35) the following kinematic field equations result:

$$\partial_0 A_0^{t_1} = \partial_k A_k^{t_1}, \quad (42)$$

$$\begin{aligned} \partial_0 A_k^{t_1} &= \partial_k A_0^{t_1} + [(\bar{m} \mu_0 + \Delta_{T t_2} \alpha_0) \delta_{t_1 t_2} \\ &\quad + (\bar{m} \mu_3 + \Delta_{T t_2} \alpha_3) (\delta_{t_1 0} \delta_{t_2 3} + \delta_{t_1 3} \delta_{t_2 0})] E_k^{t_2}, \end{aligned} \quad (43)$$

$$\begin{aligned} \partial_0 E_k^{t_1} &= -\varepsilon_{klm} \partial_l B_m^{t_1} \\ &\quad - [(\bar{m} \mu_0 + \Delta_{V t_2} \alpha_0) \delta_{t_1 t_2} + (\bar{m} \mu_3 + \Delta_{V t_2} \alpha_3) \\ &\quad \cdot (\delta_{t_1 0} \delta_{t_2 3} + \delta_{t_1 3} \delta_{t_2 0})] A_k^{t_2}, \end{aligned} \quad (44)$$

$$\partial_0 B_k^{t_1} = \varepsilon_{klm} \partial_l E_m^{t_1}. \quad (45)$$

These equations correspond to the kinematical part of vector field equations in energy representation. Compared to the field equations without symmetry breaking only the equations for  $A_k^i$  and  $E_k^i$  are modified. Due to isospin symmetry breaking a mass matrix results which depends on the values of  $\bar{m}$ ,  $\mu_0$ ,  $\mu_3$  ( $\cong$  subfermion masses  $m_1$  and  $m_2$ ),  $\alpha_0$ ,  $\alpha_3$  (degree of symmetry breaking in the propagator), and  $\Delta_{T_i}$ ,  $\Delta_{V_i}$  (depending on the coupling constants  $g_{T_i}$  and  $g_{V_i}$  for tensor and vector coupling).

As a concrete example we consider the subfermion model of Stumpf [18, 22–25]. In this model there is no tensor coupling, i.e.  $\Delta_{T_i} = 0$ . Additionally, the relevant vector coupling constants  $g_{V_v}$  (isovector) and  $g_{V_s}$  (isoscalar) are numerically identical. This yields  $\Delta_{V_i} = \Delta$ . Thus, the relevant field equations (43) and (44) for  $A_k^i$  and  $E_k^i$  explicitly read for the isospin components:

$$\begin{aligned}\partial_0 A_k^0 &= \partial_k A_0^0 + \bar{m} \mu_0 E_k^0 + \bar{m} \mu_3 E_k^3, \\ \partial_0 A_k^{1,2} &= \partial_k A_0^{1,2} + \bar{m} \mu_0 E_k^{1,2}, \\ \partial_0 A_k^3 &= \partial_k A_0^3 + \bar{m} \mu_0 E_k^3 + \bar{m} \mu_3 E_k^0,\end{aligned}\quad (46)$$

and

$$\begin{aligned}\partial_0 E_k^0 &= -\varepsilon_{klm} \partial_l B_m^0 - (\bar{m} \mu_0 + \Delta \alpha_0) A_k^0 - (\bar{m} \mu_3 + \Delta \alpha_3) A_k^3, \\ \partial_0 E_k^{1,2} &= -\varepsilon_{klm} \partial_l B_m^{1,2} - (\bar{m} \mu_0 + \Delta \alpha_0) A_k^{1,2}, \\ \partial_0 E_k^3 &= -\varepsilon_{klm} \partial_l B_m^3 - (\bar{m} \mu_0 + \Delta \alpha_0) A_k^3 - (\bar{m} \mu_3 + \Delta \alpha_3) A_k^0.\end{aligned}\quad (47)$$

In order to relate equations (46) and (47) to the standard model we introduce the linear combinations

$$A^0 \pm A^3, \quad E^0 \pm E^3, \quad B^0 \pm B^3 \quad (48)$$

and rescale the field strengths as follows:

$$\begin{aligned}E_k^{1,2} &:= \bar{m} \mu_0 E_{k/\text{old}}^{1,2}, \\ B_k^{1,2} &:= \bar{m} \mu_0 B_{k/\text{old}}^{1,2}, \\ (E_k^0 \pm E_k^3) &:= \bar{m} (\mu_0 \pm \mu_3) (E_k^0 \pm E_k^3)_{\text{old}}, \\ (B_k^0 \pm B_k^3) &:= \bar{m} (\mu_0 \pm \mu_3) (B_k^0 \pm B_k^3)_{\text{old}}.\end{aligned}\quad (49)$$

Then it follows from (46) for the potentials:

$$\begin{aligned}\partial_0 (A_k^0 + A_k^3) &= \partial_k (A_0^0 + A_0^3) + (E_k^0 + E_k^3), \\ \partial_0 (A_k^0 - A_k^3) &= \partial_k (A_0^0 - A_0^3) + (E_k^0 - E_k^3), \\ \partial_0 A_k^{1,2} &= \partial_k A_0^{1,2} + E_k^{1,2},\end{aligned}\quad (50)$$

and from (47) for the  $E$ -fields strengths:

$$\begin{aligned}\partial_0 (E_k^0 + E_k^3) &= -\varepsilon_{klm} \partial_l (B_m^0 + B_m^3) \\ &\quad - \bar{m} (\mu_0 + \mu_3) [\bar{m} (\mu_0 + \mu_3) \\ &\quad + \Delta (\alpha_0 + \alpha_3)] (A_k^0 + A_k^3),\end{aligned}$$

$$\begin{aligned}\partial_0 (E_k^0 - E_k^3) &= -\varepsilon_{klm} \partial_l (B_m^0 - B_m^3) \\ &\quad - \bar{m} (\mu_0 - \mu_3) [\bar{m} (\mu_0 - \mu_3) \\ &\quad + \Delta (\alpha_0 - \alpha_3)] (A_k^0 - A_k^3), \\ \partial_0 E_k^{1,2} &= -\varepsilon_{klm} \partial_l B_m^{1,2} - \bar{m} \mu_0 (\bar{m} \mu_0 + \Delta \alpha_0) A_k^{1,2}.\end{aligned}\quad (51)$$

From (51) we read off the mass squares

$$\begin{aligned}m_{A^0+A^3}^2 &= \bar{m} (\mu_0 + \mu_3) [\bar{m} (\mu_0 + \mu_3) \\ &\quad + \Delta (\alpha_0 + \alpha_3)] = 4m_1^2 + 2m_1 \Delta (\alpha_0 + \alpha_3),\end{aligned}\quad (52)$$

$$\begin{aligned}m_{A^0-A^3}^2 &= \bar{m} (\mu_0 - \mu_3) [\bar{m} (\mu_0 - \mu_3) \\ &\quad + \Delta (\alpha_0 - \alpha_3)] = 4m_2^2 + 2m_2 \Delta (\alpha_0 - \alpha_3),\end{aligned}\quad (53)$$

$$\begin{aligned}m_{A^{1,2}}^2 &= \bar{m} \mu_0 (\bar{m} \mu_0 + \Delta \alpha_0) \\ &= (m_1 + m_2)^2 + (m_1 + m_2) \Delta \alpha_0.\end{aligned}\quad (54)$$

Without mass correction ( $\Delta = 0$ ) we find the mass  $\bar{m} (\mu_0 + \mu_3) = 2m_1 = 2m_p$  for the state  $(A^0 + A^3) \sim p \bar{p}$ , the mass  $\bar{m} (\mu_0 - \mu_3) = 2m_2 = 2m_n$  for  $(A^0 - A^3) \sim n \bar{n}$ , and the mass  $\bar{m} \mu_0 = (m_p + m_n)/2$  for  $A^{1,2} \sim p \bar{n}$  as expected. The mass correction due to  $\mathcal{H}_1$  leads to the masses (52)–(54).

If  $A^0 + A^3$  is to be identified with the photon  $\gamma$ , the  $m_\gamma^2 = 0$  condition reads

$$2m_1 + \Delta (\alpha_0 + \alpha_3) = 0. \quad (55)$$

Then the masses for the  $Z$ -boson ( $A^0 - A^3$ ) and the  $W$ -bosons ( $A^{1,2}$ ) are given by

$$\begin{aligned}m_Z &= 2\sqrt{m_2(m_2 - m_1 - \Delta \alpha_3)}, \\ m_W &= \sqrt{(m_1 + m_2)(m_2 - m_1 - \Delta \alpha_3)},\end{aligned}\quad (56)$$

leading to the mass ratio

$$\frac{m_W}{m_Z} = \frac{1}{2} \sqrt{\frac{m_1 + m_2}{m_2}}, \quad (57)$$

which is, in this simple scheme, independent of the values of  $\alpha_0$ ,  $\alpha_3$ , and  $\Delta$ . In a more rigorous treatment these quantities should be connected by renormalization procedures. Additionally, for a realistic, quantitative subfermionic description of the electroweak theory, the effective dynamics should be evaluated with higher accuracy. This will, however, not be pursued here.

As a second example we consider the massless Heisenberg model [16] with  $\bar{m} = 0$  and  $\Delta_{T_{i2}} = 0$ . In this case a remark has to be made concerning the restriction to pure bound state dynamics which was made in our calculations. In the massive model of Stumpf the decoupling of bound state dynamics and scattering processes can be justified by very high constituent

masses and restriction to low energies [29]. In the massless model, however, this decoupling as well as the stability of bound states is not obvious and has to be checked by detailed investigations which cannot be presented here. Additionally, it has to be observed that in the massless case the weak mapping method will not provide the correct relation between the potentials  $A_\mu$  and the field strengths  $F_{\mu\nu}$  (comp. Bargmann-Wigner equations for massless fields). Instead, only the field strength equations have to be considered. For the relevant  $E$ -field equation it follows from (44):

$$\begin{aligned}\partial_0 E_k^0 &= -\varepsilon_{klm} \partial_l B_m^0 - \Delta_s \alpha_0 A_k^0 - \Delta_v \alpha_3 A_k^3, \\ \partial_0 E_k^{1,2} &= -\varepsilon_{klm} \partial_l B_m^{1,2} - \Delta_v \alpha_0 A_k^{1,2}, \\ \partial_0 E_k^3 &= -\varepsilon_{klm} \partial_l B_m^3 - \Delta_v \alpha_0 A_k^3 - \Delta_s \alpha_3 A_k^0\end{aligned}\quad (58)$$

with  $\Delta_s = \Delta_{V_s} := g_{V_s} \Delta$  and  $\Delta_v = \Delta_{V_v} := g_{V_v} \Delta$ . Now we rescale the fields according to

$$\begin{aligned}E^0 &= \sqrt{g_{V_s}} E_{\text{old}}^0, & A^0 &= \sqrt{g_{V_s}} A_{\text{old}}^0, \\ E^{1,2,3} &= \sqrt{g_{V_v}} E_{\text{old}}^{1,2,3}, & A^{1,2,3} &= \sqrt{g_{V_v}} A_{\text{old}}^{1,2,3},\end{aligned}\quad (59)$$

and from (58) the following symmetric mass matrix for  $(E^1, E^2, E^3, E^0)$  results:

$$\begin{pmatrix} \Delta_v \alpha_0 & 0 & 0 & 0 \\ 0 & \Delta_v \alpha_0 & 0 & 0 \\ 0 & 0 & \Delta_v \alpha_0 & \sqrt{\Delta_s \Delta_v} \alpha_3 \\ 0 & 0 & \sqrt{\Delta_s \Delta_v} \alpha_3 & \Delta_s \alpha_0 \end{pmatrix}. \quad (60)$$

Diagonalization of this mass matrix yields mass eigenstates and corresponding masses. We are particularly interested in mass 0 states (photon). This can only be achieved for vanishing determinant, i.e. with the condition  $\alpha_0 = \pm \alpha_3$  corresponding to maximal symmetry breaking. With  $\sqrt{g_{V_s}} = \frac{1}{2} g'$  and  $\sqrt{g_{V_v}} = \frac{1}{2} g$  it follows then for the mass matrix:

$$\frac{1}{4} \Delta \alpha_0 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & \pm g g' \\ 0 & 0 & \pm g g' & g'^2 \end{pmatrix}, \quad (61)$$

which is exactly the mass matrix of the standard Higgs mechanism if  $2 \Delta \alpha_0$  is identified with the vacuum expectation value of the Higgs field and  $g'$  and  $g$  with the  $U(1)$  and  $SU(2)$  coupling constants. Thus, we have found a microscopic description of mass generation and symmetry breaking for composite gauge bosons which is in close analogy to the standard Higgs mechanism. Together with the effective gauge interactions

which were derived in [21] this completes our effective subfermionic picture of gauge theories with isospin symmetry breaking. However, it must be emphasized again that our treatment is merely qualitative. Realistic, quantitative subfermion models of the standard model are far beyond the scope of this paper. Rather, our investigation might be seen as the sketch of a self-contained model which is structurally equivalent to the standard model without the drawbacks of the conventional Higgs mechanism.

## 5. Summary and Outlook

For the class of nonlinear spinor field models with arbitrary global  $U(1) \times SU(2)$  isospin-invariant local four-fermion self-interaction we have discussed effects of isospin symmetry breaking on resulting effective boson-fermion coupling theories with composite two-fermion bosons. To make the treatment independent of certain regularization schemes we have restricted ourselves to some general assumptions. So we have taken into account explicit isospin symmetry breaking by different masses for the isospin components, and we have expressed a non-invariant ground state in terms of a modified propagator function. (Suppressing relations between both aspects by renormalization procedures.) The evaluation of the effective boson kinematics with these assumptions showed that, in addition to the gauge field dynamics which was described in [21], the patterns of gauge boson mass generation can be reproduced. As concrete examples we chose two qualitatively distinct subfermion models, namely 1. a massive model (Stumpf) and 2. a massless theory (Heisenberg). In the first case the masses of the gauge bosons are characterized by constituent masses and binding energies, and the photon is kept massless by a finetuning process. The second model can be regarded in direct analogy to the standard Higgs model. Here, the bosons masses result from couplings to the ground state, the vacuum expectation value of the Higgs field being replaced by spinorial condensates which are expressed by the propagator function. In both models, an effective mass matrix for composite gauge bosons results which is directly related to the spinorial coupling constants.

Similar considerations were performed by Dürr and Saller [17, 30] with emphasis on the discussion of the connection between local and global symmetries and the reduction of higher symmetries to fundamental



ones. Our investigations concerning the effective composite particle dynamics support their understanding that the standard model of elementary particles may be deduced from a more fundamental spinor model in the spirit of Heisenberg's unified theory. Disregarding such ambiguous aims, the present work may indicate that nonlinear spinor theory, at least, can contribute to a better understanding of the symmetry breaking aspect in elementary particle physics (as for example in Nambu/Jona-Lasinio models of QCD). Further in-

vestigations will be devoted to the extension of the present structural and qualitative work towards a realistic subfermionic model of the standard theory.

### Acknowledgement

The author is grateful to Prof. H. Stumpf for stimulating this investigation, helpful hints during the elaboration, and for critical reading of the manuscript.

- [1] T. P. Cheng and L. F. Li, Gauge theory of elementary particle physics, Clarendon Press, Oxford 1984.
- [2] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
- [3] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
- [4] A. Salam, Weak and electromagnetic interactions. In: Elementary particle physics (N. Svartholm, ed.) (Nobel Symp. No. 8), Almquist & Wiksell, Stockholm 1968.
- [5] D. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
- [6] S. Weinberg, Phys. Rev. Lett. **31**, 494 (1973).
- [7] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. **47B**, 365 (1973).
- [8] G. 't Hooft, Nucl. Phys. **B 33**, 173 (1971).
- [9] G. 't Hooft, Nucl. Phys. **B 35**, 167 (1971).
- [10] L. Susskind, Phys. Rev. **D 20**, 2619 (1979).
- [11] S. Weinberg, Phys. Rev. **D 13**, 974 (1976).
- [12] S. Weinberg, Phys. Rev. **D 19**, 1277 (1979).
- [13] E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981).
- [14] N. N. Karpuk and S. I. Kruglov, Sov. J. Nucl. Phys. **46**, 173 (1987).
- [15] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. **D 41**, 1647 (1990).
- [16] W. Heisenberg, Introduction to the unified field theory of elementary particles. Interscience Publ., London 1966.
- [17] H. P. Dürr and H. Saller, Phys. Rev. **22**, 1176 (1980).
- [18] D. Großer, B. Hailer, L. Hornung, T. Lauxmann, and H. Stumpf, Z. Naturforsch. **38a**, 1056 (1983).
- [19] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
- [20] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961).
- [21] G. Grimm, Z. Naturforsch. **49a**, 649 (1994).
- [22] H. Stumpf, Z. Naturforsch. **40a**, 14 (1985).
- [23] H. Stumpf, Z. Naturforsch. **41a**, 683 (1986).
- [24] H. Stumpf, Z. Naturforsch. **42a**, 213 (1987).
- [25] H. Stumpf, Z. Naturforsch. **43a**, 345 (1988).
- [26] W. Pfister and H. Stumpf, Z. Naturforsch. **46a**, 389 (1991).
- [27] R. Kerschner and H. Stumpf, Ann. Phys. Leipzig **48**, 56 (1991).
- [28] W. Pfister, Nuovo Cim. **106A**, 363 (1993).
- [29] H. Stumpf and W. Pfister, Nuovo Cim. **105A**, 677 (1992).
- [30] H. Saller, Nuovo Cim. **67A**, 70 (1982).